MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

ECT1026 – FIELD THEORY

(TE/RE)

13 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **SEVEN (7) pages** including cover page and appendix with **FOUR (4) questions** only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.
4. Only non-programmable calculator is allowed.

QUESTION 1 [25 marks]

(a) On a Cartesian coordinate system, a point P located on $z-$ axis at a height of 3 units above the $x-y$ plane from an arbitrary point $Q(x, y, 4)$ at the plane $z=4$. Determine the following:

(i) Coordinate P. (1 mark)

(ii) Vector $\vec{QP}$. (2 marks)

(iii) Unit vector $\hat{QP}$. (2 marks)

(b) Determine the following vector operation if $\vec{A}=\hat{x}+2\hat{y}+3\hat{z}$, $\vec{B}=-\hat{x}-3\hat{y}-4\hat{z}$ and $\vec{C}=\hat{y}-5\hat{z}$:

(i) Component of $\vec{A}$ along $\vec{C}$ (3 marks)

(ii) θ_{AB} (3 marks)

(iii) $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$. (5 marks)

(iv) Conclude your observation in terms of direction of $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ in (iii). (2 marks)

(c) Transform vector $\vec{A}=3\hat{x}+2\hat{y}-\hat{z}$ into cylindrical coordinate system. Next, evaluate it at point $(-2, 1, 1)$. (7 marks)

Continued.....

QUESTION 2 [25 marks]

(a) State Stoke's Theorem. (3 marks)

(b) A line integral for $\vec{A} = -\hat{k}\,\frac{\sin\phi}{r}$ over the contour bounding surface which is given as $r = 3$, $\pi/3 \le \phi \le \pi/2$ and $0 \le z \le 3$ is equivalent to $\frac{\sqrt{3}}{2} - 1$. Use Stoke's Theorem to prove that the surface integral of the curl of a vector field over the stated surface is also equivalent to the given line integral value. (9 marks)

(c) Sketch the surface that has the following parameters:

$$r = 4 \quad 0 \le \phi \le \frac{\pi}{4} \qquad -3 \le z \le 3$$

Next, use the appropriate expression for the differential surface area $d\vec{S}$ to determine the area of the surface. (6 marks)

(d) An atmospheric pressure at a certain point of height of a mountain is $P(x, y, z) = z - x^2 - y^2$. A climber desires to climb in a direction such that he will feel lesser pressure. Calculate the direction he should take by finding the gradient of the pressure. (7 marks)

QUESTION 3 [25 marks]

(a) State **FOUR** categories of material classification according to the magnitudes of their conductivity. (4 marks)

(b) A positive charge of $2\ mC$ is located in a vacuum at $P(3, -2, -4)$ and a negative charge of $5\ \mu C$ is at $Q(1, -4, 2)$.

(i) Find the vector force on the negative charge. (6 marks)

(ii) Determine the electric field intensity vector caused the positive charges. (3 marks)

(iii) Calculate electric flux density vector and magnitude caused the negative charges. (6 marks)

(c) A parallel plate capacitor has an area of $5.67\times10^{-4}\ m^2$ and a plate separation of $1.99\times10^{-3}\ m$ across which a potential of $9.5\ V$ is applied. If a material having a dielectric constant of 4.59 is placed within the region between the plates, find:

(i) The capacitance. (2 marks)

(ii) The magnitude of the charge stored on each plate. (2 marks)

(iii) The dielectric displacement. (2 marks)

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QUESTION 4 [25 marks]

(a) List out the properties of magnetic materials (diamagnetism, paramagnetism and ferromagnetism) in terms of the following: (4 marks)

- Relative permeability
- Permanent magnetic

(b) Determine the magnitude of the magnetization vector for a magnetic material if it has permeability, $\mu = 1.75 \times 10^{-5}\ H/m$ and being applied an external magnetization field of $H = 130\ A/m$. (5 marks)

(c)

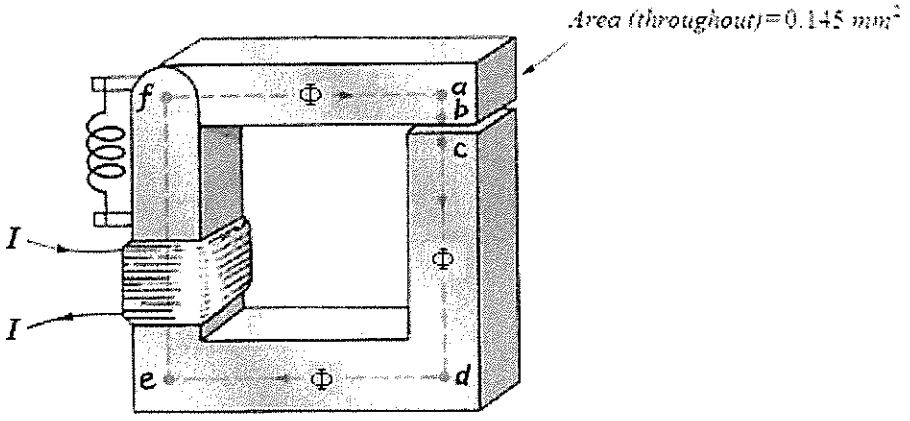

Figure 2

A series magnetic circuit as shown in **Figure 2** above is to be used to generate a magnetic flux of $\Phi = 0.75 \times 10^{-4}\ Wb$. The length of the steel $(cdefab)$, $l_{c-b} = 120\ mm$ while the length of gap (bc), $l_{b-c} = 4\ mm$. The winding number of turns, N is given as 200. Assuming that there is no flux leakage and fringing effect, determine the following:

(The relative permeability for the cast steel is $\mu_r = 1450$) Φ

(i) Magnetic flux density for core section, B_c. (2 marks)

(ii) Magnetic field intensity of the steel, H_c. (2 marks)

(iii) Magnetic field intensity of the air gap, H_g. (2 marks)

(iv) Magnetomotive force for each core section and air gap section. (4 marks)

(v) Current value to generate the magnetic flux, I. (3 marks)

(vi) Energy stored in the air gap, P. (3 marks)

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Physical Constants, Vector and Coordinate Transformation Relations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\bar{s}_x = \hat{x}dydz$ $d\bar{s}_y = \hat{y}dxdz$ $d\bar{s}_z = \hat{z}dxdy$	$d\bar{s}_r = \hat{r}rd\phi dz$ $d\bar{s}_\phi = \hat{\phi}drdz$ $d\bar{s}_z = \hat{z}rdrd\phi$	$d\bar{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\bar{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\bar{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2\sin\theta dRd\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

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Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\left(RA_\phi\right)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

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